On the Capacity of Cooperative Communication in Correlated Fading Channels

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Abstract—We investigate the performance of the protocols for cooperative communications, such as amplify-forward and decode-forward, in correlated fading channels. Our analytical and simulation results show that correlation can have varied effects on the capacity, compared to the uncorrelated cases. If the quality of the source-destination channel is better, in terms of the variances of the fading coefficients, than that of the source-relay channel, correlation may cause degradation in the capacity. Otherwise, cooperation between terminals in correlated fading channels could offer more capacity than in the uncorrelated scenarios.

Keywords—capacity; cooperative communication; correlation

I. INTRODUCTION

Recently, cooperative communications through exploiting the virtual antenna array constituted by distributed radio terminals has drawn considerable attention from researchers [1]-[4]. Sendonaris et. al have initially demonstrated in [1] that with the help of a relay node the transmission performance between the source and destination nodes can be enhanced. An overview of this promising technology was given in [4].

The cooperative protocols at the medium access control (MAC) layer have been presented and categorized into three types [3]: fixed relaying, selection relaying, and incremental relaying. In fixed relaying, relay nodes are always required. In selection relaying, relay nodes work only when the quality of the source-relay channel is above an expectation level. In incremental relaying, the involvement of relay nodes depends on the feedback from the destination node. For each of the above protocols, there exist two modes: amplify-forward and decode-forward. The former simply scales the received analogue signal before relaying, but at the same time the receiver noise will be amplified as well. The latter involves fully or symbol-by-symbol source information decoding.

So far, cooperative communication has been studied mainly in the uncorrelated fading channels. It is therefore essential to know the actual benefits that could be achieved using this technology in a more realistic environment, such as in correlated fading channels. In this paper, we intend to examine the capacity for cooperative communications through correlated fading channels. First, we derive the correlated channel model and the variances of the correlated fading coefficients, with the presence of correlation coefficient and the variances of the uncorrelated fading channels. Then we analyze the effect of correlation on the capacity for fixed amplify-forward and selection decode-forward, which will also be illustrated through simulations.

The paper is organised as follows. In Section II, we discuss the protocols used in our work. We then examine the variances of the correlated fading channels in Section III, which will be used to analyze the capacity in Section IV. Afterwards, we show some simulation results in Section V that illustrate the capacity in line with the analysis given previously. Finally, we draw our conclusions in Section VI.

II. SYSTEM MODEL AND PROTOCOLS

In this paper, we consider a system comprising only one relay node together with one source and one destination nodes. In this system, each node involved in transmission is designated to work in its own time slots. The time scheduling of the system consists of two phases, as shown in Fig. 1. Here, we apply the two-phase model to two protocols: fixed amplify-forward and selection decode-forward, without loss of generality. The capacity for the two protocols is given below [3]

\[ C_{AF} = \frac{1}{2} \log_2 \left( 1 + \xi \left( |\alpha_{rs}|^2 + \frac{2|\alpha_{sr}|^2}{|\alpha_{rs}|^2 + |\alpha_{dr}|^2 + 1} \right) \right), \quad (1) \]

\[ C_{DF} = \begin{cases} \frac{1}{2} \log_2 \left( 1 + \xi \cdot \frac{2|\alpha_{sr}|^2}{|\alpha_{rs}|^2 + |\alpha_{dr}|^2} \right) & \text{if } |\alpha_{rs}|^2 < g(\xi), \\ \frac{1}{2} \log_2 \left( 1 + \xi \cdot \frac{2|\alpha_{sr}|^2}{|\alpha_{rs}|^2 + |\alpha_{dr}|^2} \right) & \text{if } |\alpha_{rs}|^2 \geq g(\xi) \end{cases} \quad (2) \]
where $a_{ij}, i \in \{s, r\}, j \in \{r, d\}$ denote the complex fading coefficients of the individual channels in the system, $\sim \mathcal{CN}(0, \sigma^2); g(\zeta) = (2^{2\beta} - 1) / \zeta$, $R$ is the spectral efficiency attempted by the source, $\zeta$ is the signal-to-noise ratio (SNR) attempted by the source and the relay nodes, assumed to be the same at both.

In the second phase of fixed amplify-forward, only the relay destination channel transmits, while the source-destination channel may be occupied again if the relay node cannot involve in selection decode-forward.

### III. CORRELATED FADING CHANNELS

Referring to Fig. 1 (a), the source node is the transmitter and both the relay and destination nodes are the receivers, so correlation exists between the two channels: the source-destination channel and the source-relay channel. The scale of this correlation is related to the location of the relay node and the scattering environment. For instance, a relay node being situated on the line between the source and the destination nodes can cause considerable amount of correlation, which will rise when the relay node gets closer to the destination. In Phase II, either the relay-destination channel or the source-relay channel is involved, which means no correlation occurs inside the system during this phase.

Since a cooperative system is more likely to be asymmetric in terms of the variances of the wireless channels, the classic channel model, $B^{1/2}H_{\text{IND}}A^{1/2}$, cannot be used in correlated channel modelling in such a system (IID stands for independent and nonidentically distributed). Instead, we utilize the following approach which considers a multiple-antenna channel with $n_T$ transmit and $n_R$ receive antennas, described by the model as in [5]

$$H = U_s H_{\text{IND}} U_r^*,$$

where $U_R$ and $U_T$ are $n_R \times n_R$ and $n_T \times n_T$ deterministic unitary matrices; $^*$ stands for complex transpose conjugate; the entries of $H_{\text{IND}}$ (IND stands for independent and nonidentically distributed) are zero mean and independent with arbitrary marginal distributions and variance, constrained only to

$$\mathbb{E}[T[H_{\text{IND}}H_{\text{IND}}^*]] = n_s n_r.$$

This $H_{\text{IND}}$ can be regarded as a channel matrix with independent nonidentically distributed entries. We assume that [5, eq. (19)] is upheld. Therefore, the columns of $U_R$ and $U_T$ correspond to the eigenvectors of $\mathbb{E}[HH^*]$ and $\mathbb{E}[H^*H]$, respectively. The variances of the entries in $H_{\text{IND}}$ are the eigenvalues of the covariance matrix, $Q$, of $H$. We aim, in this paper, to model this $H$ in the cooperative communication environment which takes account of correlation in Phase I.

The channel matrix (vector) $H$ of Phase I with correlation can be expressed as

$$H = \begin{bmatrix} \alpha_{sr} & \alpha_{sd} \\ \alpha_{dr} & \alpha_{dd} \end{bmatrix} = U_s \begin{bmatrix} \alpha_{srr} \\ \alpha_{srd} \end{bmatrix} U_r^*,$$

where $\alpha$ represents the fading coefficients on all the channels in both correlated and uncorrelated cases.

Since $H$ is a vector, $\mathbb{E}[HH^*]$ is actually the covariance matrix of $H$, which is modelled as follows

$$Q = \mathbb{E}[HH^*] = \begin{bmatrix} \sigma_{ss}^2 & \sigma_{sr}^2 \\ \sigma_{sr}^2 & \sigma_{rr}^2 \end{bmatrix},$$

where $\sigma$ is the standard deviation for all the channels in both correlated and uncorrelated cases, and $\beta$ and $\theta$ are the amplitude and angle of the complex correlation coefficients between the source-destination and the source-relay channels. From (5), we can work out its eigenvalues which will be the variances of the entries in $H_{\text{IND}}$ and the relative eigenvectors which will be used to construct $U_R$.

Through the standard process of calculating eigenvalues, the eigenvalues of $Q$ are found as

$$\lambda = [\sigma_{ss}^2, \sigma_{rr}^2].$$

From (6), we can present the variances of the entries of $H$, $\sigma_{ss}^2$ and $\sigma_{rr}^2$, in terms of the correlation amplitude, $\beta$, and the variances of the entries of $H_{\text{IND}}, \sigma_{sr}^2$ and $\sigma_{dr}^2$,

$$\sigma_{ss}^2 = \frac{1}{2} \left( 4\sigma_{ss}^2 \sigma_{dr}^2 - \left( \sigma_{sr}^2 + \sigma_{ss}^2 \right)^2 + 4\sigma_{sr}^2 \sigma_{dr}^2 - \left( \sigma_{sr}^2 + \sigma_{rr}^2 \right)^2 \right),$$

$$\sigma_{rr}^2 = \frac{1}{2} \left( 4\sigma_{rr}^2 \sigma_{sr}^2 - \left( \sigma_{sr}^2 + \sigma_{rr}^2 \right)^2 + 4\sigma_{sr}^2 \sigma_{sr}^2 - \left( \sigma_{sr}^2 + \sigma_{ss}^2 \right)^2 \right),$$

$$\sigma_{sr}^2 = \frac{1}{2} \left( 4\sigma_{sr}^2 \sigma_{dr}^2 - \left( \sigma_{sr}^2 + \sigma_{ss}^2 \right)^2 + 4\sigma_{sr}^2 \sigma_{dr}^2 - \left( \sigma_{sr}^2 + \sigma_{rr}^2 \right)^2 \right).$$
where $\beta \in [0, 1)$. For (9) and (10) to be real values, the following condition has to be fulfilled that

$$
\max(\sigma_{s,d}^2, \sigma_{s,d}^2) \cdot 1 + \beta \geq \min(\sigma_{s,d}^2, \sigma_{s,d}^2) \cdot 1 - \beta \tag{11}
$$

Eq. (9) and (10) show the two facts: first, if $\sigma_{s,d}^2 > \sigma_{s,d}^2$, then it follows that $\sigma_{s,r}^2 > \sigma_{s,r}^2$, and vice versa; secondly, given $\sigma_{s,d}^2$ and $\sigma_{s,d}^2$ and assume $\sigma_{s,d}^2 > \sigma_{s,d}^2$, they will be changed in two opposite directions when the two channels become correlated, i.e., $\sigma_{s,r}^2 < \sigma_{s,r}^2$ and $\sigma_{s,r}^2 > \sigma_{s,r}^2$. In other words, the quality of one of the two channels is degraded while the other is improved. Comparing $(\sigma_{s,d}^2, \sigma_{s,d}^2)$ and $(\sigma_{s,d}^2, \sigma_{s,d}^2)$, we discover that

$$
\Delta_{\sigma} = \frac{|\sigma_{s,d}^2 - \sigma_{s,d}^2| - |\sigma_{s,d}^2 - \sigma_{s,d}^2|}{\sqrt{2}} \tag{12}
$$

which shows that the correlation affects the quality in terms of variance of the two channels by the same scale.

### IV. CAPACITY IN CORRELATED FADING CHANNELS

With the effect of correlation, the quality of the source-destination and source-relay channels can be degraded or improved depending on the conditions of the two channels. In this section, we investigate how the capacity of the two protocols discussed in this paper varies with the changed channel quality due to correlation.

A. Fixed Amplify-Forward

Having taken correlation into account, the capacity (1) should be altered to, for simulation,

$$
C_{DF} = \frac{1}{2} \log_2 \left[ 1 + \frac{\beta \sigma_{s,r}^2}{|\sigma_{s,d}^2 + \sigma_{s,d}^2 + 1|} \right] \tag{13}
$$

Since correlation has impacts on the source-relay channel and the source-destination channel symmetrically, we modify (13) for the correlated channels to, for analysis,

$$
C_{DF} = \frac{1}{2} \log_2 \left[ 1 + \frac{\beta \sigma_{s,d}^2}{|\sigma_{s,d}^2 + \sigma_{s,d}^2 + 1|} \right] \tag{14}
$$

where $\Delta_{\sigma}$ is the absolute variation amount of fading coefficients caused by correlation, and $\gamma$ is the coefficient between $[0, 1)$. Eq. (14) stands because for example, it is true that $(m + \Delta_{\sigma}) n/(m + \Delta_{\sigma} + n + 1) = m n/(m + n + 1) + \gamma \Delta_{\sigma}$.

### TABLE I. SIMULATION CONFIGURATIONS

<table>
<thead>
<tr>
<th>$\sigma_{s,d}^2$</th>
<th>$\sigma_{s,d}^2$</th>
<th>$\sigma_{s,d}^2$</th>
<th>$\beta$</th>
<th>$R$ (bits/s/Hz)</th>
<th>$SNR$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-D Better</td>
<td>0.1</td>
<td>0.6</td>
<td>0.9</td>
<td>0.7</td>
<td>1</td>
</tr>
<tr>
<td>S-R Better</td>
<td>0.6</td>
<td>0.1</td>
<td>0.9</td>
<td>0.4</td>
<td>20</td>
</tr>
</tbody>
</table>

When $\sigma_{s,d}^2 \geq \sigma_{s,d}^2$, the extent of capacity variation is subject to $(1 - \gamma)\Delta_{\sigma}$ (degradation), while it depends on $(1 - \gamma)\Delta_{\sigma}$ (improvement) for $\sigma_{s,d}^2 \leq \sigma_{s,d}^2$.

B. Selection Decode-Forward

The capacity of this protocol is modified to, with the presence of correlation,

$$
C_{DF} = \frac{1}{2} \log_2 \left[ 1 + \frac{\beta \sigma_{s,r}^2}{|\sigma_{s,d}^2 + \sigma_{s,d}^2 + 1|} \right] \tag{15}
$$

Comparing (2) and (15), we observe that the variation of the capacity due to correlation highly depends on the quality of the source-destination channel, which is affected by correlation. When $\sigma_{s,d}^2 \geq \sigma_{s,d}^2$, the capacity is expected to decrease, while the channel performs better when $\sigma_{s,d}^2 \leq \sigma_{s,d}^2$.

Shown by (15), the improvement of the capacity comes from the re-transmission on the source-destination channel, and the relay node involves less when the quality of the correlated source-relay channel drops due to correlation.

Therefore, for both protocols, the capacity is determined mainly by the quality of the source-destination channel in correlation scenario.

V. SIMULATION RESULTS DISCUSSION

The capacity and outage probability of the correlated cooperative systems are illustrated in Fig. 2 & 3, according to the simulation configurations tabulated in TABLE I. The results are generated through 10000 Monte-Carlo simulations. The cooperative system can be simulated by either (4) or (9) and (10). Here, we adopt the latter. The first set of configurations, “S-D Better”, represents the scenario that the source-relay channel is in an environment with many signal impairment factors, while the source-destination is in better conditions. The second configuration shows the scenario discussed in the first paragraph of Section III. In both configurations, the variance of the relay-destination channel is fixed to 0.9. Also, we assume the channels are Rayleigh faded.

In the “S-D Better” scenario, correlation impairs the capacity for both protocols. However, in the “S-R Better” environment, cooperative systems achieve capacity benefits in all the data regions by using both fixed amplify-forward and selection decode-forward protocols.

The differences in the capacity between the uncorrelated systems and the correlated ones rise along with the increased amount of correlation. This is because the variation amount of the variances caused by correlation increases when the amplitude of correlation, $\beta$, rises.
In “S-D Better” scenario, the source-destination channel is degraded with higher correlation. Therefore, the capacity for both protocols decreases, which is rather explicit in the high correlation scenario, shown by Fig. 2. For fixed amplify-forward, \((-1+\gamma)\Delta_\alpha\) introduces loss; while for selection decode-forward, although the relay node is more involved, the capacity still decreases due to the impaired source-destination channel.

Fig. 3 for “S-R Better” scenario has shown performance improvement in association with correlation. In fixed amplify-forward, correlation displays its positive effect over the entire capacity span. For selection decode-forward, the improvement increases as the amplitude of correlation coefficients becomes greater. This is because the relay node involves less with higher correlation and the quality of the source-destination channel is improved by correlation, leading to higher capacity.

VI. CONCLUSIONS

We first derived the correlated channel model and the variances of the correlated fading coefficients, in terms of correlation coefficient and the variances of the uncorrelated channels. We then analyze the effects of correlation on the capacity for the fixed amplify-forward protocol and the selection decode-forward protocol. It has been shown by the simulation results that the differences in quality between the source-destination channel and the source-relay channel lead to different capacity results in the correlated environment. We have seen that a better source-relay channel condition relative to that of the source-destination channel is the key for higher capacity when cooperative communications takes place in correlated fading channel. Otherwise, the capacity is impaired when the relationship between the two channel conditions is other way around.

REFERENCES