Chirped solitons with strong confinement in transmission links with in-line fiber Bragg gratings

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We demonstrate that the use of fiber gratings to reverse periodically the pulse chirp in transmission links allows us to produce stable chirped optical solitons with fast-decaying tails. The interaction between neighboring pulses is reduced from that of the usual soliton system as a result of pulse chirping and strong confinement, making possible dense information packing. Such a pulse is an attractive candidate for use as an information carrier in optical transmission systems with ultralarge capacities of approximately 100 Gbits/s.

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In this Letter we demonstrate that incorporation of in-line Bragg gratings for a periodic reverse of the pulse chirp makes it possible to produce a chirped carrier pulse with strong confinement (and, consequently, with suppressed interaction) even without use of the rather complex dispersion profile suggested in Ref. 5. We show that the quasi-soliton described in Ref. 5 presents a limiting case of the usual dispersion-managed soliton, and all the remarkable properties of a quasi-soliton can be obtained with simple dispersion management obtainable by use of gratings. Additionally, our approach gives simple rules for optimizing the performance of fiber transmission systems similar to those considered in Refs. 6–8.

The dynamics of the central part of the dispersion-managed soliton is given in the leading order by (for details see Ref. 3 and references therein)

$$\Psi(z, t) = \frac{Q(t/T)}{\sqrt{T}} \exp(i \frac{M}{T} t^2 + i \lambda),$$

$$\frac{dT}{dz} = 4d(z)M, \quad \frac{dM}{dz} = \frac{C_1d}{T^3} - \frac{c(z)C_2}{T^2},$$

where $$c(z) = P_0L\sigma \exp[2L \int_0^z (-\gamma + r \sum_{k=1}^N \delta(z' - z_k))dz'];$$ $$\gamma$$ is normalized to a dispersion map length $$L;$$ time and power are normalized to scaling parameters $$t_0$$ and $$P_0,$$ respectively (in the following illustrations $$t_0 = 5$$ ps and $$P_0 = 2$$ mW); $$\sigma$$ is the nonlinear coefficient; $$\gamma$$ describes fiber losses; $$r$$ is the amplification coefficient; $$z_k$$ are the amplifiers’ locations; the normalized dispersion is $$d(z) = -L\beta_2/(2t_0^2),$$ where $$\beta_2$$ is the group-velocity dispersion that varies periodically with $$z;$$ and the constants $$C_1$$ and $$C_2$$ are related to the pulse shape $$Q(x)$$ through $$C_1 = \int |Q_x|^2dx/\int x^2|Q|^2dx$$ and $$C_2 = \int |Q|^2dx/\int x^2|Q|^2dx.$$ The lumped action of the amplifiers and the gratings is accounted for through a pulse parameter transformation at junctions that correspond to locations of amplifiers and gratings. The point action of the gratings can be described...
in Fourier space as $\Psi_{\text{out}}(t) = \int_{-\infty}^{\infty} \exp(-i\omega^2 - i\omega t) \Psi(\omega) d\omega$, where $\Psi(\omega)$ is a Fourier image of the pulse before the gratings. The effect of a grating can easily be understood for a chirped Gaussian pulse $\Psi_{\text{in}}(t) = A_{\text{in}} \exp(-t^2/(2T_{\text{in}}^2)) + it^2M_{\text{in}}/T_{\text{in}}$. In this case the linear grating operator $g$ transforms an input pulse into $\Psi_{\text{out}}(t) = g \Psi_{\text{in}} = A_{\text{out}} \exp(-t^2/(2T_{\text{out}}^2)) + it^2M_{\text{out}}/T_{\text{out}}$. When the condition $g = -2M_{\text{in}}T_{\text{in}}^5/(1 + 4M_{\text{in}}^2T_{\text{in}}^2)$ holds, then $T_{\text{out}} = T_{\text{in}}, M_{\text{out}} = -M_{\text{in}}, |A_{\text{out}}| = |A_{\text{in}}|$, and therefore the grating is tuned to hold the pulse width unchanged, reversing the chirp. Although the main results of this research are formulated here in a general form and can be used for many dispersion maps, to make the idea clearest in the illustrations without loss of generality we consider the simplest possible dispersion-managed system, using chirped gratings (see, e.g., Refs. 6–8). A line is built from the dispersion-shifted fibers with anomalous dispersion $D = 1$ ps/(nm × km); gratings are placed with the period $L = 100$ km.

To avoid mathematical details we formulate the main idea briefly and verify results by direct numerical simulations of the basic model described in detail in many publications. As was shown,\textsuperscript{3,4} the slow evolution of the central (energy-containing) part of the pulse propagating in a system with strong dispersion management is described by the nonlinear Schrödinger equation with an additional parabolic potential that varies as $a t^2$, where $a = \langle \bar{a}(z) \rangle = \langle T(z)M_z \rangle = (C_1 r_1 - C_2 r_2)$, $r_1 = \langle \bar{d}/T^2 \rangle$ and $r_2 = \langle c/T \rangle$. $T$ is a solution of Eqs. (1). $C_1$ and $C_2$ are defined above, and the angle brackets mean averaging over a period $L$. The sign of $a$ plays a crucial role in the dynamics of the DMS. A positive $a$ would provide well-confined solutions with Gaussian tails (as in the case of a quasi-soliton), whereas a negative $a$ corresponds to the solutions with oscillatory tails. As was proved in Ref. 4, $a$ is always negative for true periodic solutions of Eqs. (1) and corresponds to the nontrapping effective potential and to the tunneling of the radiation from the central part of the DMS. Note that, even for a quasi-soliton, $a$ defined as above is negative. However, the instantaneous potential $\bar{a}(z)$ is always positive (for a quasi-soliton), which results in strong confinement of the quasi-soliton and superior characteristics for soliton interaction. The main observation of the present Letter is that, though for periodic solutions of Eqs. (1) the parameter $a$ is always negative, by simple dispersion management it is possible to make $\bar{a}(z)$ be always positive within the compensation section. This result leads to formation of a stable chirped pulse with strong confinement and suppressed interaction. In our scheme the attractive features of the quasi-soliton can be obtained with simple dispersion management, in contrast to the relatively complex dispersion maps considered in Ref. 5. Note that the far-field tails are still exponential, but, because of the fast decay of the intensity in the central part of the solitons, their amplitude is less than that of the tails of a fundamental soliton with the same peak power (and the same local dispersion). As a matter of fact, we show also that the quasi-soliton described in Ref. 5 is a particular limiting case of a DMS.

The results of our numerical simulations are presented in Figs. 1–4. In Fig. 1 we compare the evolution of the soliton chirp and the pulse width found by direct simulations and by solution of Eqs. (1). Coefficients $C_1$ and $C_2$ are determined from the pulse shape, as mentioned above. Figure 2 shows the evolution of the chirped soliton along one section. The pulse

![Fig. 1. Comparison of the evolution of (top) the soliton chirp and (bottom) the pulse width found by direct simulation of the partial differential equation (PDE) and by solution of ordinary differential equation (1).](image1)

![Fig. 2. Evolution of the chirped soliton during one compensation period.](image2)
is compressed in the middle by the effects of non-linearity and initial prechirping and recovers its width at the end of the cell. Then the grating reverses the chirp of the pulse and the soliton phase is recovered. Figure 3 displays the shape of the chirped soliton compared with those of sech-shaped solitons with the same peak power and with the same pulse width, corresponding to the local dispersion in the section. The left-hand inset of Fig. 3 shows that, as expected, the chirped pulse has enhanced power compared with that of a sech-shaped soliton of the same width, corresponding to the same path-average dispersion. Figure 4 illustrates the reduced interaction of chirped solitons. Conventional solitons of the same pulse width and initial separation (corresponding to the same local dispersion) collapse after passing through 10 sections.

In conclusion, we have found that the use of fiber Bragg gratings to reverse a pulse chirp allows us (with simple dispersion management) to produce a chirped carrier pulse with strong confinement, which results in substantial suppression of soliton interaction and a possibility for dense information packing. Such a pulse is an attractive candidate for use as an information carrier in optical transmission systems with ultralarge capacities of approximately 100 Gbits/s.

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References